

4. 4. 誤差逆伝播法

c) 隠れ層3層の計算

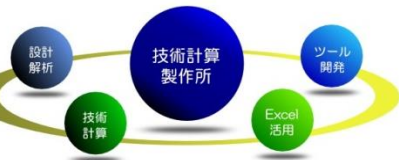
$\frac{\partial L}{\partial W^1}$ の計算を行います。この計算ではb) で求めた (4.4-10) 式をそのまま使用します。

$$\frac{\partial L}{\partial W^1} = \zeta X^t \frac{\partial b}{\partial Y^1} \{g^1(A^1)\}' \quad \dots (4.4 - 10)$$

また、 $\frac{\partial b}{\partial Y^1}$ の計算結果は (4.4-11) 式の $(W^2)^t$ を $\frac{\partial A^3}{\partial Y^1}$ に書き換えるだけです。

$$\frac{\partial b}{\partial Y^1} = (W^4)^t \{g^3(A^3)\}' \frac{\partial A^3}{\partial Y^1} \quad \dots (4.4 - 13)$$

あとは $\frac{\partial A^3}{\partial Y^1}$ を計算すれば $\frac{\partial L}{\partial W^1}$ が求まります。この計算も成分表示で行います。



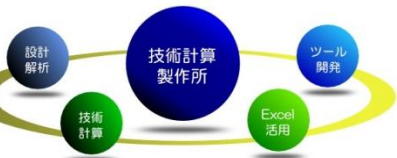
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$$\begin{aligned}\frac{\partial a_1^3}{\partial y_1^1} &= \frac{\partial}{\partial y_1^1} (y_1^2 w_{11}^3 + y_2^2 w_{21}^3) \\ &= w_{11}^3 \frac{\partial y_1^2}{\partial a_1^2} \frac{\partial a_1^2}{\partial y_1^1} + w_{21}^3 \frac{\partial y_2^2}{\partial a_2^2} \frac{\partial a_2^2}{\partial y_1^1} \\ &= w_{11}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{11}^2 + w_{21}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{12}^2\end{aligned}$$

$$\begin{aligned}\frac{\partial a_1^3}{\partial y_2^1} &= \frac{\partial}{\partial y_2^1} (y_1^2 w_{11}^3 + y_2^2 w_{21}^3) \\ &= w_{11}^3 \frac{\partial y_1^2}{\partial a_1^2} \frac{\partial a_1^2}{\partial y_2^1} + w_{21}^3 \frac{\partial y_2^2}{\partial a_2^2} \frac{\partial a_2^2}{\partial y_2^1} \\ &= w_{11}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{21}^2 + w_{21}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{22}^2\end{aligned}$$

$$\begin{aligned}\frac{\partial a_2^3}{\partial y_1^1} &= \frac{\partial}{\partial y_1^1} (y_1^2 w_{12}^3 + y_2^2 w_{22}^3) \\ &= w_{12}^3 \frac{\partial y_1^2}{\partial a_1^2} \frac{\partial a_1^2}{\partial y_1^1} + w_{22}^3 \frac{\partial y_2^2}{\partial a_2^2} \frac{\partial a_2^2}{\partial y_1^1} \\ &= w_{12}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{11}^2 + w_{22}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{12}^2\end{aligned}$$

$$\begin{aligned}\frac{\partial a_2^3}{\partial y_2^1} &= \frac{\partial}{\partial y_2^1} (y_1^2 w_{12}^3 + y_2^2 w_{22}^3) \\ &= w_{12}^3 \frac{\partial y_1^2}{\partial a_1^2} \frac{\partial a_1^2}{\partial y_2^1} + w_{22}^3 \frac{\partial y_2^2}{\partial a_2^2} \frac{\partial a_2^2}{\partial y_2^1} \\ &= w_{12}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{21}^2 + w_{22}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{22}^2\end{aligned}$$



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$$\begin{aligned} \frac{\partial Y^3}{\partial A^1} &= \begin{pmatrix} w_{11}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{11}^2 + w_{21}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{12}^2 & w_{11}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{21}^2 + w_{21}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{22}^2 \\ w_{12}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{11}^2 + w_{22}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{12}^2 & w_{12}^3 \frac{\partial y_1^2}{\partial a_1^2} w_{21}^2 + w_{22}^3 \frac{\partial y_2^2}{\partial a_2^2} w_{22}^2 \end{pmatrix} \\ &= \begin{pmatrix} w_{11}^3 & w_{21}^3 \\ w_{12}^3 & w_{22}^3 \end{pmatrix} \begin{pmatrix} \{g^2(a_1^2)\}' & 0 \\ 0 & \{g^2(a_2^2)\}' \end{pmatrix} \begin{pmatrix} w_{11}^2 & w_{21}^2 \\ w_{12}^2 & w_{22}^2 \end{pmatrix} \\ &= (W^3)^t \{g^2(A^2)\}' (W^2)^t \quad \dots (4.4 - 12) \end{aligned}$$

これを (4.4-11) 式に代入して、 $\frac{\partial b}{\partial Y^1}$ が求まります。

$$\frac{\partial b}{\partial Y^1} = (W^4)^t \{g^3(A^3)\}' (W^3)^t \{g^2(A^2)\}' (W^2)^t \quad \dots (4.4 - 14)$$

さらにこれを (4.4-10) 式に代入して、 $\frac{\partial L}{\partial W^1}$ が求まります。

$$\begin{aligned} \frac{\partial L}{\partial W^1} &= X^t \frac{\partial L}{\partial z} \{h(b)\}' (W^4)^t \{g^3(A^3)\}' (W^3)^t \{g^2(A^2)\}' (W^2)^t \{g^1(A^1)\}' \quad \dots (4.4 - 15) \\ &\left(\zeta = \frac{\partial L}{\partial z} \{h(b)\}' \right) \end{aligned}$$

