4．4．誤差逆伝播法

c）隠れ層3層の計算

$\frac{\partial L}{\partial W^1}$の計算を行います。この計算ではb）で求めた（4.4-10）式をそのまま使用します。

$$\frac{\partial L}{\partial W^1} = \zeta X^t \frac{\partial b}{\partial Y^1}\{g^1(A^1)\}' \quad \cdots (4.4-10)$$

また、$\frac{\partial b}{\partial Y^1}$の計算結果は（4.4-11）式の$(W^2)^t$を$\frac{\partial A^3}{\partial Y^1}$に書き換えるだけです。

$$\frac{\partial b}{\partial Y^1} = (W^4)^t\{g^3(A^3)\}'\frac{\partial A^3}{\partial Y^1} \quad \cdots (4.4-13)$$

あとは$\frac{\partial A^3}{\partial Y^1}$を計算すれば$\frac{\partial L}{\partial W^1}$が求まります。この計算も成分表示で行います。

4. 4. 誤差逆伝播法

$$
\begin{aligned}
\frac{\partial {a_1}^3}{\partial {y_1}^1} &= \frac{\partial}{\partial {y_1}^1}\left({y_1}^2{w_{11}}^3 + {y_2}^2{w_{21}}^3\right) \\
&= {w_{11}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}\frac{\partial {a_1}^2}{\partial {y_1}^1} + {w_{21}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}\frac{\partial {a_2}^2}{\partial {y_1}^1} \\
&= {w_{11}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}{w_{11}}^2 + {w_{21}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}{w_{12}}^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial {a_1}^3}{\partial {y_2}^1} &= \frac{\partial}{\partial {y_2}^1}\left({y_1}^2{w_{11}}^3 + {y_2}^2{w_{21}}^3\right) \\
&= {w_{11}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}\frac{\partial {a_1}^2}{\partial {y_2}^1} + {w_{21}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}\frac{\partial {a_2}^2}{\partial {y_2}^1} \\
&= {w_{11}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}{w_{21}}^2 + {w_{21}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}{w_{22}}^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial {a_2}^3}{\partial {y_1}^1} &= \frac{\partial}{\partial {y_1}^1}\left({y_1}^2{w_{12}}^3 + {y_2}^2{w_{22}}^3\right) \\
&= {w_{12}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}\frac{\partial {a_1}^2}{\partial {y_1}^1} + {w_{22}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}\frac{\partial {a_2}^2}{\partial {y_1}^1} \\
&= {w_{12}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}{w_{11}}^2 + {w_{22}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}{w_{12}}^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial {a_2}^3}{\partial {y_2}^1} &= \frac{\partial}{\partial {y_2}^1}\left({y_1}^2{w_{12}}^3 + {y_2}^2{w_{22}}^3\right) \\
&= {w_{12}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}\frac{\partial {a_1}^2}{\partial {y_2}^1} + {w_{22}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}\frac{\partial {a_2}^2}{\partial {y_2}^1} \\
&= {w_{12}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2}{w_{21}}^2 + {w_{22}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2}{w_{22}}^2
\end{aligned}
$$

4. 4. 誤差逆伝播法

$$\frac{\partial Y^3}{\partial A^1} = \begin{pmatrix} {w_{11}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2} {w_{11}}^2 + {w_{21}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2} {w_{12}}^2 & {w_{11}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2} {w_{21}}^2 + {w_{21}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2} {w_{22}}^2 \\ {w_{12}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2} {w_{11}}^2 + {w_{22}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2} {w_{12}}^2 & {w_{12}}^3 \frac{\partial {y_1}^2}{\partial {a_1}^2} {w_{21}}^2 + {w_{22}}^3 \frac{\partial {y_2}^2}{\partial {a_2}^2} {w_{22}}^2 \end{pmatrix}$$

$$= \begin{pmatrix} {w_{11}}^3 & {w_{21}}^3 \\ {w_{12}}^3 & {w_{22}}^3 \end{pmatrix} \begin{pmatrix} \{g^2({a_1}^2)\}' & 0 \\ 0 & \{g^2({a_2}^2)\}' \end{pmatrix} \begin{pmatrix} {w_{11}}^2 & {w_{21}}^2 \\ {w_{12}}^2 & {w_{22}}^2 \end{pmatrix}$$

$$= (W^3)^t \{g^2(A^2)\}' (W^2)^t \quad \cdots (4.4-12)$$

これを（4.4-11）式に代入して、$\frac{\partial b}{\partial Y^1}$が求まります。

$$\frac{\partial b}{\partial Y^1} = (W^4)^t \{g^3(A^3)\}' (W^3)^t \{g^2(A^2)\}' (W^2)^t \quad \cdots (4.4-14)$$

さらにこれを（4.4-10）式に代入して、$\frac{\partial L}{\partial W^1}$が求まります。

$$\frac{\partial L}{\partial W^1} = X^t \frac{\partial L}{\partial z} \{h(b)\}' (W^4)^t \{g^3(A^3)\}' (W^3)^t \{g^2(A^2)\}' (W^2)^t \{g^1(A^1)\}' \quad \cdots (4.4-15)$$

$$\left(\zeta = \frac{\partial L}{\partial z} \{h(b)\}' \right)$$