

$$\begin{aligned} \therefore (D_1 D_2 - D_2 D_1) f = \\ \sum \left\{ \left( \frac{\partial g}{\partial q_i} \frac{\partial^2 h}{\partial p_i \partial q_k} - \frac{\partial g}{\partial p_i} \frac{\partial^2 h}{\partial q_i \partial q_k} - \frac{\partial h}{\partial q_i} \frac{\partial^2 g}{\partial p_i \partial q_k} + \frac{\partial h}{\partial p_i} \frac{\partial^2 g}{\partial q_i \partial q_k} \right) \frac{\partial f}{\partial p_k} \right. \\ \left. - \left( \frac{\partial g}{\partial q_i} \frac{\partial^2 h}{\partial p_i \partial p_k} - \frac{\partial g}{\partial p_i} \frac{\partial^2 h}{\partial q_i \partial p_k} - \frac{\partial h}{\partial q_i} \frac{\partial^2 g}{\partial p_i \partial p_k} + \frac{\partial h}{\partial p_i} \frac{\partial^2 g}{\partial q_i \partial p_k} \right) \frac{\partial f}{\partial q_k} \right\} \dots (5.2.3-3) \end{aligned}$$

さらに、(8) 式左辺第一項について

$$\{f\{gh\}\} = \sum \left\{ \frac{\partial f}{\partial g_i} \frac{\partial}{\partial p_i} \left( \frac{\partial g}{\partial q_\nu} \frac{\partial h}{\partial p_\nu} - \frac{\partial g}{\partial p_\nu} \frac{\partial h}{\partial q_\nu} \right) - \frac{\partial f}{\partial p_i} \frac{\partial}{\partial q_i} \left( \frac{\partial g}{\partial q_\nu} \frac{\partial h}{\partial p_\nu} - \frac{\partial g}{\partial p_\nu} \frac{\partial h}{\partial q_\nu} \right) \right\}$$

$$\begin{aligned}
&= \sum \left\{ \frac{\partial f}{\partial q_i} \frac{\partial}{\partial p_i} \left( \frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} \right) - \frac{\partial f}{\partial q_i} \frac{\partial}{\partial p_i} \left( \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_k} \right) - \frac{\partial f}{\partial p_i} \frac{\partial}{\partial q_i} \left( \frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} \right) + \frac{\partial f}{\partial p_i} \frac{\partial}{\partial q_i} \left( \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_k} \right) \right\} \\
\therefore \quad \{f\{gh\}\} &= \sum \left\{ - \left( \frac{\partial h}{\partial p_k} \frac{\partial^2 g}{\partial q_i \partial q_k} - \frac{\partial g}{\partial q_k} \frac{\partial^2 h}{\partial q_i \partial p_k} + \frac{\partial h}{\partial q_k} \frac{\partial^2 g}{\partial q_i \partial p_k} + \frac{\partial g}{\partial p_k} \frac{\partial^2 h}{\partial q_i \partial q_k} \right) \frac{\partial f}{\partial p_i} \right. \\
&\quad \left. + \left( \frac{\partial g}{\partial q_k} \frac{\partial^2 h}{\partial p_i \partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial^2 h}{\partial p_i \partial q_k} - \frac{\partial h}{\partial q_k} \frac{\partial^2 g}{\partial p_i \partial p_k} + \frac{\partial h}{\partial p_k} \frac{\partial^2 g}{\partial p_i \partial q_k} \right) \frac{\partial f}{\partial q_i} \right\} \dots (5.2.3 - 4)
\end{aligned}$$